

Course Code: MAT206
Course Name: GRAPH THEORY

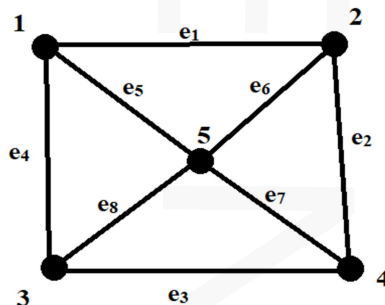
Max. Marks: 100

Duration: 3 Hours

PART A*(Answer all questions; each question carries 3 marks)*

Marks

- | | | |
|---|--|---|
| 1 | What is the maximum number of edges in a simple graph with n vertices? Justify your answer. | 3 |
| 2 | There are 25 telephones in Metropolis. Is it possible to connect them with wires so that each telephone is connected with exactly 7 others? Why? | 3 |
| 3 | Show that all vertices of an Euler graph G are of even degree | 3 |
| 4 | Explain strongly connected and weakly connected graphs with the help of examples. | 3 |
| 5 | Prove that a connected graph G with n vertices and $n-1$ edges is a tree. | 3 |
| 6 | How many labelled trees are there with n vertices? Draw all labelled trees with 3 vertices. | 3 |
| 7 | Define planar graphs. Is K_4 , the complete graph with 4 vertices, a planar graph? Justify. | 3 |
| 8 | Define fundamental circuits and fundamental cut-sets. | 3 |
| 9 | Construct the adjacency matrix and incidence matrix of the graph . | 3 |



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| 10 | Define chromatic number. What is the chromatic number of a tree with two or more vertices? | 3 |
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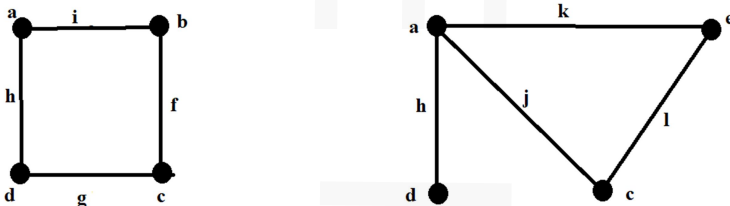
PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

- 11 a) Define complete graph and complete bipartite graph. Draw a graph which is a complete graph as well as a complete bipartite graph. 7
- b) Explain walks, paths and circuits with the help of examples. 7
- 12 a) Define isolated vertex, pendant vertex, even vertex and odd vertex. Draw a graph that contains all the above. 7
- b) Prove that simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges. 7

Module -2

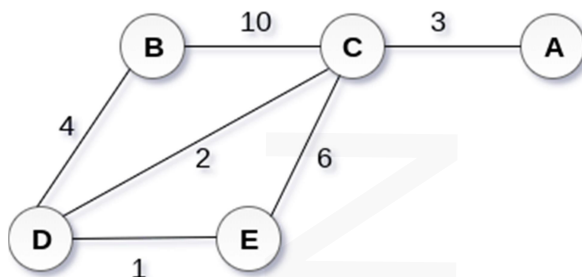
- 13 a)  9

Find the union, intersection and ring sum of the above graphs.

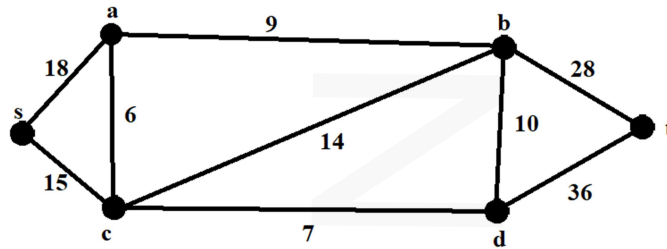
- b) State travelling salesman problem. How it is related to Hamiltonian circuits? 5
- 14 a) Prove that in a complete graph with n vertices there are $(n-1)/2$ edge disjoint Hamiltonian circuits, if n is an odd number and $n \geq 3$. 7
- b) For which values of m, n is the complete graph $K_{m,n}$ an Euler graph? Justify your answer. 7

Module -3

- 15 a) Prove that a binary tree with n vertices has $(n+1)/2$ pendant vertices. 7
- b) Using Prim's algorithm, find a minimal spanning tree for the following graph. 7



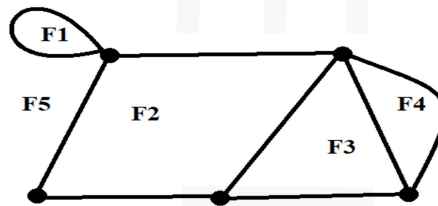
- 16 a) Write down Dijkstra's algorithm and use it to find the shortest path from s to t . 9



- b) Prove that every tree has either one or two centers. 5

Module -4

- 17 a) Define cut-set. Prove that every circuit in G has an even number of edges in common with any cut-set. 8
- b) Construct the geometric dual of the graph below 6



- 18 a) Prove that a connected planar graph with n vertices and e edges has $e-n+2$ regions. 9
- b) Let G be a connected graph and e an edge of G . Show that e is a cut-edge if and only if e belongs to every spanning tree. 5

Module -5

- 19 a) Explain *four colour problem* using the concept of chromatic number. 5
- b) Let B and A be the circuit matrix and the incidence matrix of a graph G which is free from loops, whose columns are arranged using the same order of edges. Show that $AB^T = BA^T = 0 \pmod{2}$. 9
- 20 a) Show that chromatic polynomial of a tree with n vertices is $P_n(\lambda) = \lambda(\lambda - 1)^{n-1}$ 7
- b) Define path matrix of a graph. Find the path matrix $P(x, y)$ for the graph below. 7

